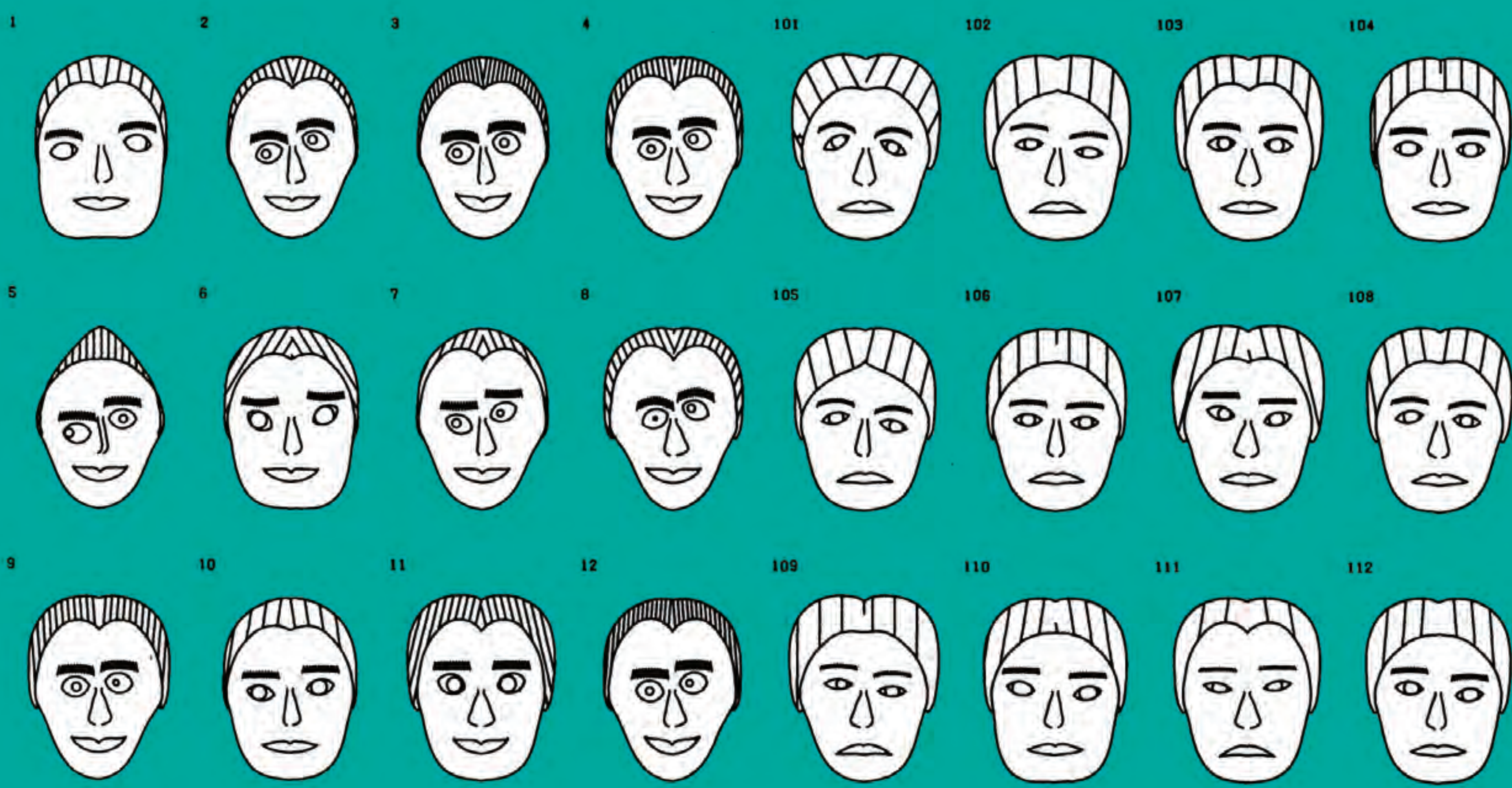
SPECIAL ISSUE

ANNALS OF
IMPROBABLE RESEARCH

MATHEMATICS



Chernoff Faces, Horse Calculus, Cocker's Arithmetick...

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Horse Calculus

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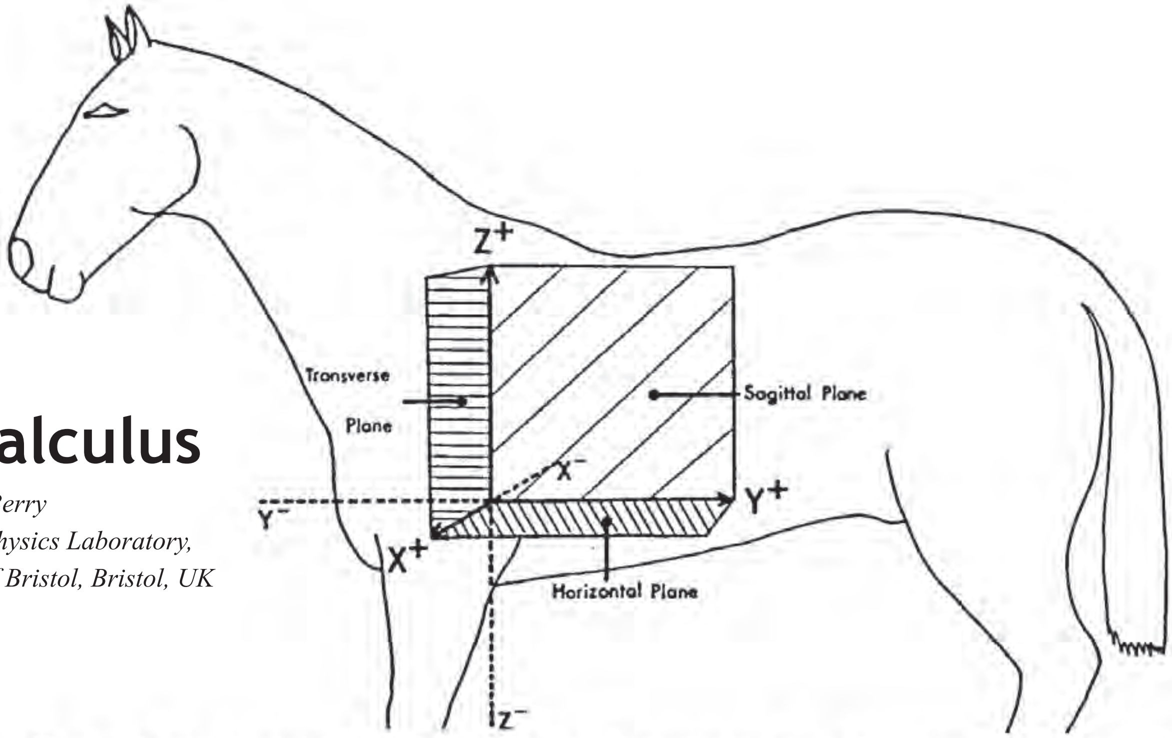
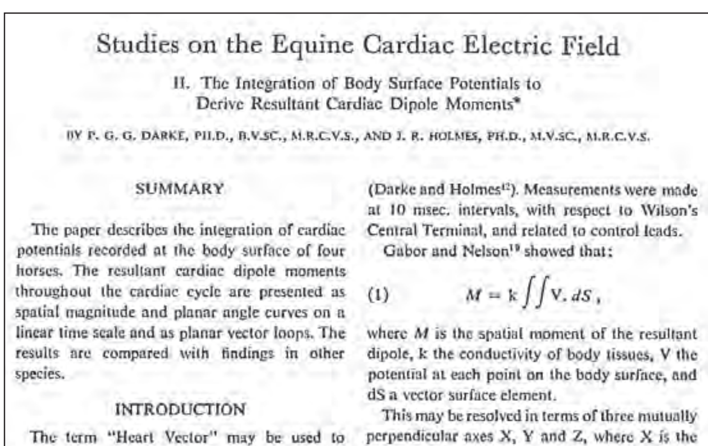


Fig. 1. The three orthogonal axes, *X* (transverse), *Y* (longitudinal) and *Z* (vertical) and their relevant planes.

The applications of mathematics can be bizarre. Soon after I arrived in Bristol in the 1960s, a senior colleague called me, saying that someone in the veterinary school needed help with mathematics — or was it physics? — and I seemed just the person to help. Cursing inwardly, I agreed to see the fellow. He was Peter Darke, a graduate student near the end of a Ph.D. studying horses' hearts.

He showed me a paper by Gabor (Dennis Gabor, who invented holography) and Nelson[1] and asked me to explain it. It took a while to understand. The idea is that a heart is like a little battery, pushing weak electric currents in a three-dimensional pattern round the body. The battery has a strength and a direction: it acts as a current dipole, represented as a little arrow — the heart vector. During each heartbeat, the vector (tip of the arrow) draws a loop - the heart loop — whose shape is a powerful diagnostic of health. Therefore it is useful to measure this loop, in a way that doesn't involve killing the horse. Gabor's paper gave the theory of a way to do that, inferring the heart vector by measurements of the electric potential on the surface of the horse. It is an ingenious application of Gauss's theorem.



Studies on the Equine Cardiac Electric Field

II. The Integration of Body Surface Potentials to Derive Resultant Cardiac Dipole Moments*

BY P. G. G. DARKE, PH.D., B.V.SC., M.R.C.V.S., AND J. R. HOLMES, PH.D., M.V.SC., M.R.C.V.S.

SUMMARY

The paper describes the integration of cardiac potentials recorded at the body surface of four horses. The resultant cardiac dipole moments throughout the cardiac cycle are presented as spatial magnitude and planar angle curves on a linear time scale and as planar vector loops. The results are compared with findings in other species.

INTRODUCTION

The term "Heart Vector" may be used to

(Darke and Holmes[12]). Measurements were made at 10 msec. intervals, with respect to Wilson's Central Terminal, and related to control leads.

Gabor and Nelson[19] showed that:

(1) $$M = k \iint V . dS ,$$

where M is the spatial moment of the resultant dipole, k the conductivity of body tissues, V the potential at each point on the body surface, and dS a vector surface element.

This may be resolved in terms of three mutually perpendicular axes X, Y and Z, where X is the

Peter had spent three years preparing to implement this idea. He enveloped his horse in a coat he had made, of several hundred potentiometers, with electronics to measure the potential at each of them, fifteen times during each heartbeat, and he had arrived at the point where he had a huge file of all these measurements. But there was a difficulty: he knew only the most elementary high-school mathematics and so had no way to understand the formulas in Gabor's paper. His specific question was: does the theory apply to a real horse, or only to an ideal cylindrical horse? Unlike the physicists' mythical 'spherical cow,' this was real.

I learned that the formulas work for a horse of any shape, but they do assume uniform conductivity — a better approximation, apparently, for horses than for people. (Actually, it doesn't have to be accurate: who cares whether the loop describes the real dipole inside the real horse? To be useful for diagnosis, it is necessary only that the loop be reproducible.)

TOP: Detail from the Darke/Holmes study.

LEFT: The Darke/Holmes study, which used the Berry approach to integrate over the surface of a horse.

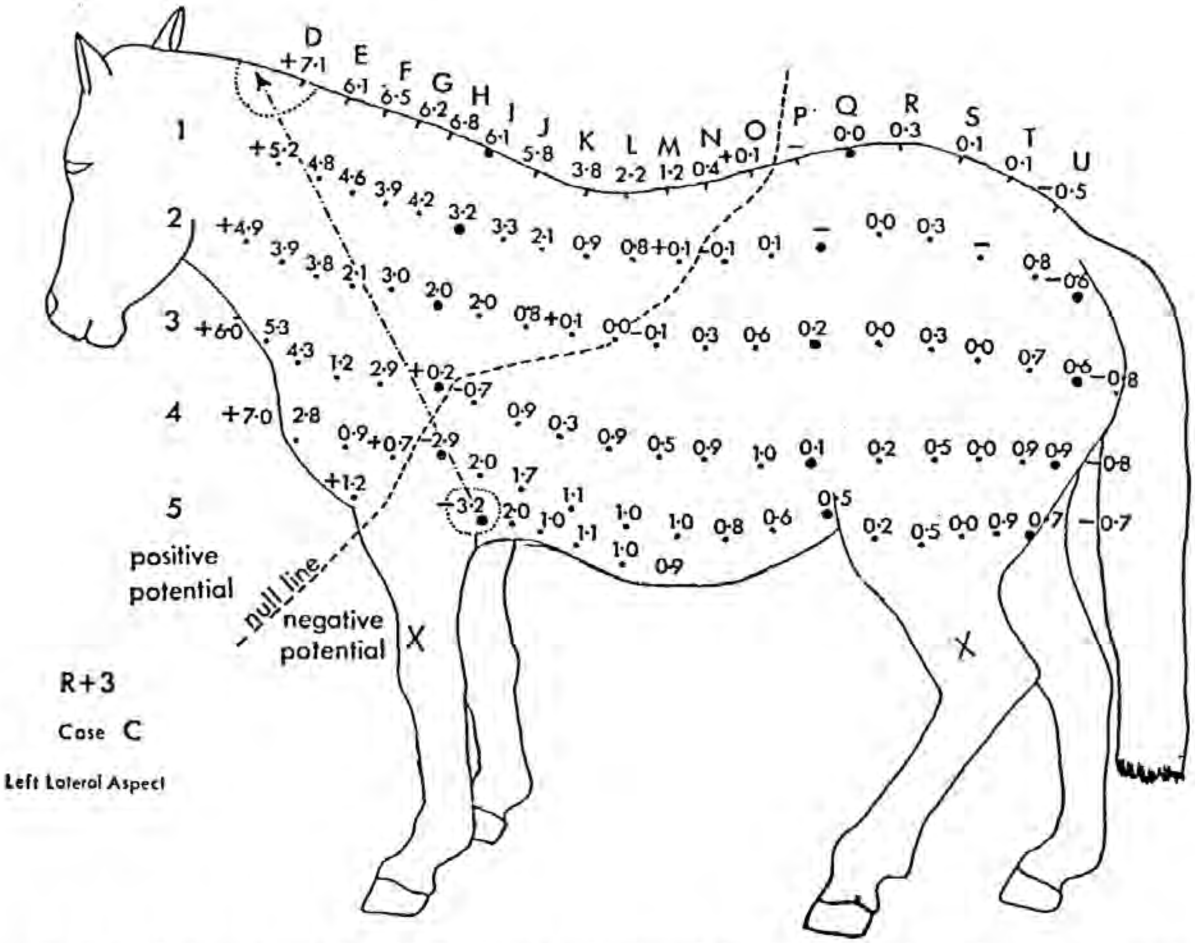


Fig. 1. Left side of horse (Case C) illustrating positioning of electrodes. The values refer to the potentials at the sites in terms of mm. of deflection (mV/10) at 30 msec. after the peak of the R wave in the reference lead. The arrow depicts the vector axis constructed between the points of maximum positive and negative potential. *All* potentials in "positive potential area" in this figure and Fig. 2 are +, similarly *all* potentials in the "negative potentials area" are −. Sites given a sign but no value (e.g. "−") represent values definitely negative but < 0.1 mm (mV/10).

Further detail from the Darke/Holmes study.

The formulas involved integration, and Peter didn't know what an integral was, so it was hard to explain how to add up all those measurements. A complication was that what had to be inferred was a vector, so he needed to know, at each point on the horse, the components of the perpendicular to the surface of the horse with respect to the three symmetry directions of the horse. After some discussion, we made a 'cos-theta-meter,' and I left him to it, and never saw him again.

But a year later, I received two papers from him,[2] reporting the outcome of all that arithmetic. To my surprise, he had indeed calculated fifteen vectors for each heartbeat, and thereby deduced the heart loops for several horses in different states of health. At the end of the paper were the usual acknowlegements to colleagues and funding agencies. For technical help, he thanked me; and for financial support, he thanked the Horserace Betting Levy Board (financed by racecourse gamblers).

The moral of this is that applications of mathematical knowledge can be unexpected; you may find yourself taking a surface integral over a horse.

References

1. "Determination of the Resultant Dipole of the Heart from Measurements on the Heart Surface," D. Gabor and C.V. Nelson, *Journal of Applied Physics*, vol. 25, 1954, pp. 413-6.

2. "Studies on the Equine Cardiac Electric Field. I. Body Surface Potentials, II. The Integration of Body Surface Potentials to Derive Resultant Cardiac Dipole Moments," P.G.G. Darke and J.R. Holmes, *Journal of Electrocardiology* vol. 2, 1969, pp. 222-234 and 235-244.